Constructive canonicity for lattice-based fixed point logics

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What is constructive canonicity?

Preservation of validity of inequalities under (constructive) canonical extensions:

$$\mathbb{A} \models \varphi \leq \psi \quad \Rightarrow \quad \mathbb{A}^\delta \models \varphi \leq \psi.$$

(Constructive) canonical extension of lattice $\mathbb{A}$

Complete lattice $\mathbb{A}^\delta$ containing $\mathbb{A}$ as a dense and compact sublattice

In the presence of the Axiom of Choice, $\mathbb{A}^\delta$ is perfect:

- $J^\infty(\mathbb{A}^\delta)$ is completely join-dense in $\mathbb{A}^\delta$, and
- $M^\infty(\mathbb{A}^\delta)$ is completely meet-dense in $\mathbb{A}^\delta$.

In the constructive setting: not enough join/meet-irreducibles

Our results

[Conradie Craig 2014]: canonicity for mu-calculus

- distributive-based, with fixed points, specific signature
- non-constructive metatheory

[Conradie Palmigiano 2016]: constructive canonicity

- general lattice-based, no fixed points, arbitrary signature
- constructive metatheory

[CCPZ16]: constructive canonicity for lattice-based fixed point logics

- general lattice-based, with fixed points, arbitrary signature
- constructive metatheory
- smooth and modular extension, supported by the unified correspondence approach

A general strategy of canonicity via ALBA

$$
\begin{array}{ccc}
\mathbb{A} \models \alpha \leq \beta & & \mathbb{A}^\delta \models \alpha \leq \beta \\
\Updownarrow & & \\
\mathbb{A}^\delta \models_{\mathbb{A}} \alpha \leq \beta & & \Big\Updownarrow \\
\Updownarrow & & \\
\mathbb{A}^\delta \models_{\mathbb{A}} \text{ALBA}(\alpha \leq \beta) & \Longleftrightarrow & \mathbb{A}^\delta \models \text{ALBA}(\alpha \leq \beta)
\end{array}
$$

We apply this strategy to lattice-based logics with fixed points

Two interpretations of fixed point operators

Motivation: completeness

Problem: canonical extension changes the values of fixed point formulas

In the lattice expansion $\mathbb{A}$:

$$\mu x.t(x, a_1, \ldots, a_{n-1}) := \bigwedge\{ a \in A \mid t(a, a_1, \ldots, a_{n-1}) \leq a \}$$

if this meet exists, otherwise $\mu x.t(x, a_1, \ldots, a_{n-1})$ is undefined.

In the canonical extension $\mathbb{A}^\delta$ of lattice expansion $\mathbb{A}$:

$$\mu^* x.t(x, a_1, \ldots, a_{n-1}) := \bigwedge\{ a \in A \mid t(a, a_1, \ldots, a_{n-1}) \leq a \}$$

Consequence: two definitions of canonicity

Two definitions of canonicity

$\varphi \leq \psi$ is canonical:

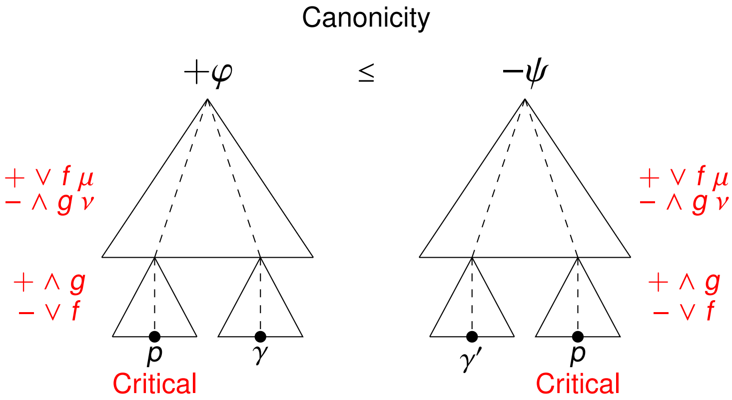
$$\mathbb{A} \models \varphi \leq \psi \quad \Rightarrow \quad \mathbb{A}^\delta \models \varphi \leq \psi.$$

$\varphi \leq \psi$ is tame canonical:

$$\mathbb{A} \models \varphi \leq \psi \quad \Rightarrow \quad \mathbb{A}^\delta \models \varphi^* \leq \psi^*.$$

Two Syntactic Characterizations

From the two notions of canonicity, two syntactic characterizations arise of formulas guaranteed to be canonical for each type:



Two Syntactic Characterizations

From the two notions of canonicity, two syntactic characterizations arise of formulas guaranteed to be canonical for each type:

Tame canonicity