Minimal and Subminimal Logic of Negation

A Sequent Calculus System

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[1] This is a Master's thesis project carried out at the ILLC under the supervision of Marta Bílková and Dick de Jongh

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Language

- $\mathcal{L}^{\neg} = \mathcal{L}^{+} \cup \{\neg\}$, where $\mathcal{L}^{+} = \{\wedge, \vee, \rightarrow\}$

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Introduction

Heying's Axioms

- Positive axioms
- $(p \to q) \wedge (p \to \neg q) \to \neg p$
- $p \wedge \neg p \to q$

Introduction

Heyting's Axioms

- Positive axioms
- $(p \to q) \wedge (p \to \neg q) \to \neg p$
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Johansson's Axioms

- Positive axioms
- $(p \to q) \wedge (p \to \neg q) \to \neg p$

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Setting

- Positive fragment of intuitionistic propositional logic

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Introduction

Axioms of Negation

- $(p \leftrightarrow q) \rightarrow (\neg p \leftrightarrow \neg q)$
- $p \wedge \neg p \rightarrow \neg q$
- $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$
- $(p \rightarrow \neg p) \rightarrow \neg p$
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Kripke-style Semantics

Kripke Frames

$$\mathfrak{F} = \langle W, R, N \rangle$$

- Intuitionistic Kripke frame $\mathfrak{F} = \langle W, R \rangle$
- Function $N : \mathcal{U}(W) \to \mathcal{U}(W)$ such that

$$\boxed{N(U) \cap V = N(U \cap V) \cap V}$$

Kripke-style Semantics

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- Intuitionistic Kripke frame $\mathfrak{F} = \langle W, R \rangle$
- Function $N : \mathcal{U}(W) \rightarrow \mathcal{U}(W)$ such that

$$w \in N(U) \text{ if and only if } w \in N(U \cap R(w))$$

Kripke-style Semantics

Results & Applications

- Semantic Clause for Negation

$$\mathfrak{M}, w \vDash \neg\varphi \iff w \in N(V(\varphi))$$

- Completeness (via Canonical Model)
- Finite Model Property
- Disjunction Property

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Sequent Calculus Systems

Rules for the Positive Fragment of Intuitionistic Logic

From system G3m, Section 3.4 of A. S. Troelstra, H. Schwitchenberg, '*Basic Proof Theory*'

$$\mathsf{Ax}\quad \Gamma, p \Rightarrow p \qquad\qquad \top\mathsf{R}\quad \Gamma \Rightarrow \top$$

$$\wedge\mathsf{L}\ \frac{\Gamma, \alpha, \beta \Rightarrow \varphi}{\Gamma, \alpha \wedge \beta \Rightarrow \varphi} \qquad\qquad \wedge\mathsf{R}\ \frac{\Gamma \Rightarrow \alpha \qquad \Gamma \Rightarrow \beta}{\Gamma \Rightarrow \alpha \wedge \beta}$$

$$\vee\mathsf{L}\ \frac{\Gamma, \alpha \Rightarrow \varphi \qquad \Gamma, \beta \Rightarrow \varphi}{\Gamma, \alpha \vee \beta \Rightarrow \varphi} \qquad\qquad \vee\mathsf{R}\ \frac{\Gamma \Rightarrow \alpha_i}{\Gamma \Rightarrow \alpha_1 \vee \alpha_2}\ , i = 1, 2$$

$$\to\mathsf{L}\ \frac{\Gamma, \alpha \to \beta \Rightarrow \alpha \qquad \Gamma, \beta \Rightarrow \varphi}{\Gamma, \alpha \to \beta \Rightarrow \varphi} \qquad \to\mathsf{R}\ \frac{\Gamma, \alpha \Rightarrow \beta}{\Gamma \Rightarrow \alpha \to \beta}$$

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$$\wedge R \frac{\Gamma \Rightarrow \alpha \qquad \Gamma \Rightarrow \beta}{\Gamma \Rightarrow \alpha \wedge \beta}$$

$$\vee L \frac{\Gamma, \alpha \Rightarrow \varphi \qquad \Gamma, \beta \Rightarrow \varphi}{\Gamma, \alpha \vee \beta \Rightarrow \varphi}$$

$$\vee R \frac{\Gamma \Rightarrow \alpha_i}{\Gamma \Rightarrow \alpha_1 \vee \alpha_2}, i = 1, 2$$

$$\rightarrow L \frac{\Gamma, \alpha \rightarrow \beta \Rightarrow \alpha \qquad \Gamma, \beta \Rightarrow \varphi}{\Gamma, \alpha \rightarrow \beta \Rightarrow \varphi}$$

$$\rightarrow R \frac{\Gamma, \alpha \Rightarrow \beta}{\Gamma \Rightarrow \alpha \rightarrow \beta}$$

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Sequent Calculus Systems

Rules for the ¬ Operator

$$\text{N}\ \frac{\Gamma, \neg\alpha, \alpha \Rightarrow \beta \qquad \Gamma, \neg\alpha, \beta \Rightarrow \alpha}{\Gamma, \neg\alpha \Rightarrow \neg\beta}$$

$$\text{NeF}\ \frac{\Gamma, \neg\alpha \Rightarrow \alpha}{\Gamma, \neg\alpha \Rightarrow \neg\beta}$$

$$\text{CoPC}\ \frac{\Gamma, \neg\alpha, \beta \Rightarrow \alpha}{\Gamma, \neg\alpha \Rightarrow \neg\beta}$$

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Rules for the $\neg$ Operator

$$\mathsf{N}\ \frac{\Gamma, \neg\alpha, \alpha \Rightarrow \beta \qquad \Gamma, \neg\alpha, \beta \Rightarrow \alpha}{\Gamma, \neg\alpha \Rightarrow \neg\beta}$$

$$\mathsf{NeF}\ \frac{\Gamma, \neg\alpha \Rightarrow \alpha}{\Gamma, \neg\alpha \Rightarrow \neg\beta}$$

$$\mathsf{CoPC}\ \frac{\Gamma, \neg\alpha, \beta \Rightarrow \alpha}{\Gamma, \neg\alpha \Rightarrow \neg\beta}$$

$$\mathsf{An}\ \frac{\Gamma, \alpha \Rightarrow \neg\alpha}{\Gamma \Rightarrow \neg\alpha}$$

Sequent Calculus Systems

Results & Applications

- Contraction and Weakening admissibility
- Cut elimination
- Cut-free proofs search
- Decidability
- Craig's Interpolation Theorem
- Translation

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Cut-free Proofs

$$\neg\neg\neg p \rightarrow \neg p$$

Applications

Cut-free Proofs

$$\neg\neg\neg p \to \neg p$$

$$\neg\neg\neg p \Rightarrow \neg p$$

Applications

Cut-free Proofs

$$\frac{\neg\neg\neg p, p \Rightarrow \neg\neg p}{\neg\neg\neg p \Rightarrow \neg p}$$

Applications

Cut-free Proofs

$$\frac{\neg\neg\neg \boldsymbol{p}, p \Rightarrow \neg\neg \boldsymbol{p}}{\neg\neg\neg p \Rightarrow \neg p}$$

$$\dfrac{\dfrac{\neg\neg\neg p, p, \neg p \Rightarrow \neg\neg p}{\neg\neg\neg p, p \Rightarrow \neg\neg p}}{\neg\neg\neg p \Rightarrow \neg p}$$

Applications

Cut-free Proofs

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Cut-free Proofs

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Applications

Translation from MPC into CoPC

We define a translation $\varphi^\sim$ by recursion over the complexity of φ:

- $p^\sim := p$
- $\top^\sim := \top$
- $(\varphi \circ \psi)^\sim := \varphi^\sim \circ \psi^\sim$, where $\circ \in \{\wedge, \vee, \to\}$
- $(\neg\varphi)^\sim := \varphi^\sim \to \neg\varphi^\sim$

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Further Work

- Further Axioms
- Terminating Sequent Calculi
- Weakened **LC**

Thank you!

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